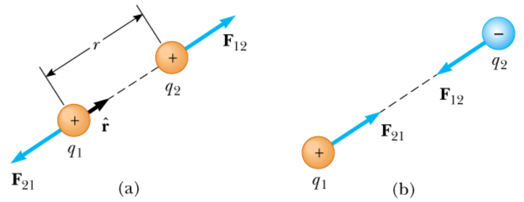
Electrostatics

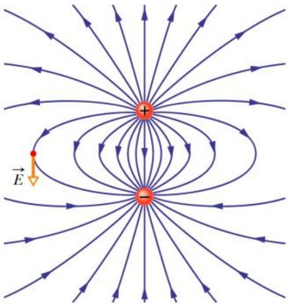
$$\vec{\mathbf{F}}_{12} = k_e \frac{q_1 q_2}{r^2}\hat{\mathbf{r}}$$

Attractive/repulsive force between charges



$$\vec{\mathbf{E}} = k_e \frac{q}{r^2}\hat{\mathbf{r}}$$

Every charge generates an electric field



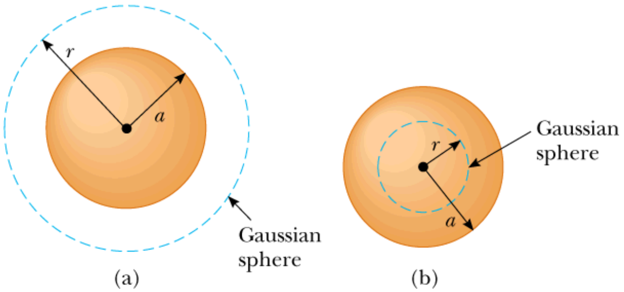
$$\vec{\mathbf{F}}_{\mathbf{e}} = q\vec{\mathbf{E}}$$

$$\Phi_E = \int_{surface} \vec{\mathbf{E}}.\mathbf{d}\vec{\mathbf{A}}$$

Electric Flux

$$\Phi_E = \oint \vec{\mathbf{E}}.d\vec{\mathbf{A}} = \frac{q_{in}}{\varepsilon_0}$$

Gauss' Law



Only charges within the Gaussian surface contribute to the electric flux through the surface

Electric Potential

Electric Potential Energy

$$\Delta U = -q_0 \int_A^B \vec{\mathbf{E}}.d\vec{\mathbf{s}}$$

Potential Difference

$$\Delta V = \frac{\Delta U}{q_0} = -\int_A^B \vec{\mathbf{E}}.d\vec{\mathbf{s}}$$

In a Uniform Field

$$\Delta V = -Ed$$

For a Point Charge

$$V = k_e \frac{q}{r}$$

For a Charge Distribution

$$V = k_e \int \frac{dq}{r}$$

$$E_x = \frac{\partial V}{\partial x}$$

$$E_y = \frac{\partial V}{\partial y}$$

$$E_z = \frac{\partial V}{\partial z}$$

Conductors in Electric Fields

- There is no electric field inside a conductor.
- The field on the surface is perpendicular to the surface and proportional to the surface charge density.
- All charges reside on the surface.
- Electric potential is constant throughout the conductor.

Capacitors

$$C \equiv \frac{Q}{\Delta V}$$

(C/V=F)

Parallel Plate Capacitor

$$C = \frac{Q}{\Delta V} = \frac{\varepsilon_0 A}{d}$$

For series capacitors

$$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}$$

For parallel capacitors

$$C_{eq} = \sum_{j=1}^{n} C_j$$

Stored Energy in a Capacitor

$$U = \frac{Q^2}{2C} = \frac{1}{2} Q\Delta V = \frac{1}{2} C(\Delta V)^2$$

Capacitor with a dielectric

$$C = \kappa C_0$$

DC Currents and Resistive Circuits

$$I(t) = \frac{dQ}{dt}$$

$$\vec{\mathbf{J}} = \sigma \vec{\mathbf{E}}$$

$$R = \frac{\Delta V}{I}$$

$$R = \rho \frac{l}{A}$$

$$\rho = \frac{1}{\sigma}$$

$$\mathcal{P} = I\Delta V = I^2 R = \frac{(\Delta V)^2}{R}$$

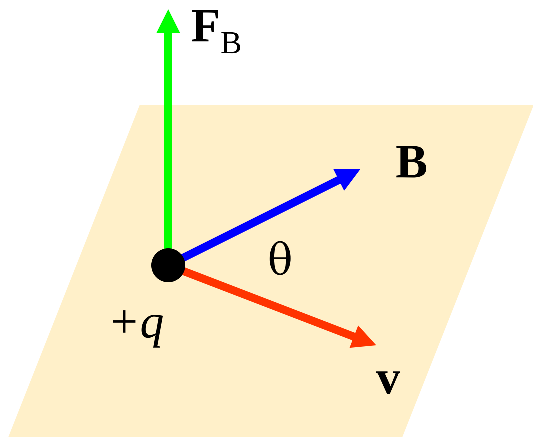
Kirchoff's Rules

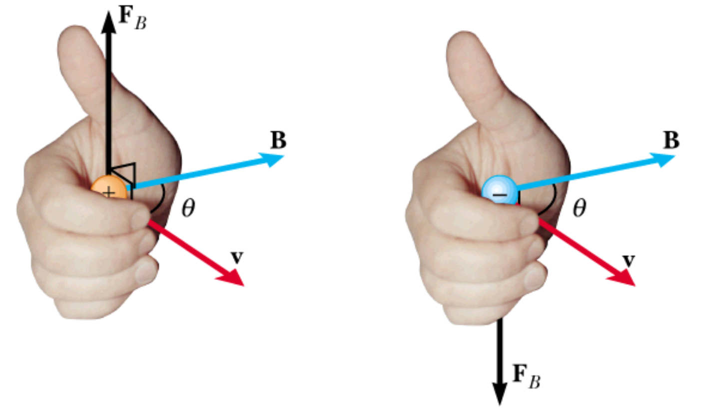
$$\sum_{\substack{closed \\ loop}} \Delta V = 0$$

$$\sum_{junction} I_{in} = \sum_{junction} I_{out}$$

Magnetic Force

$$\vec{F}_B = q\vec{v}\times\vec{B}$$

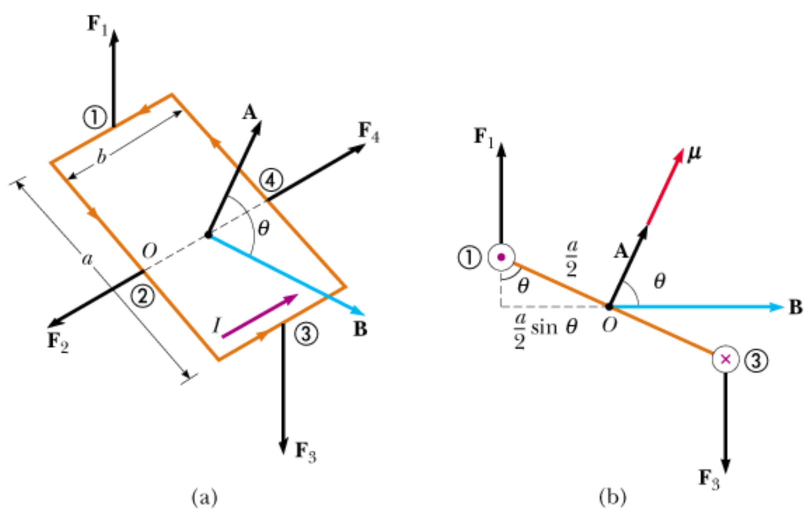




$$\vec{F}_B = I\vec{L}\times\vec{B}$$

For a straight wire in a uniform field

For a closed loop of current



$$\vec{F}_B = 0$$

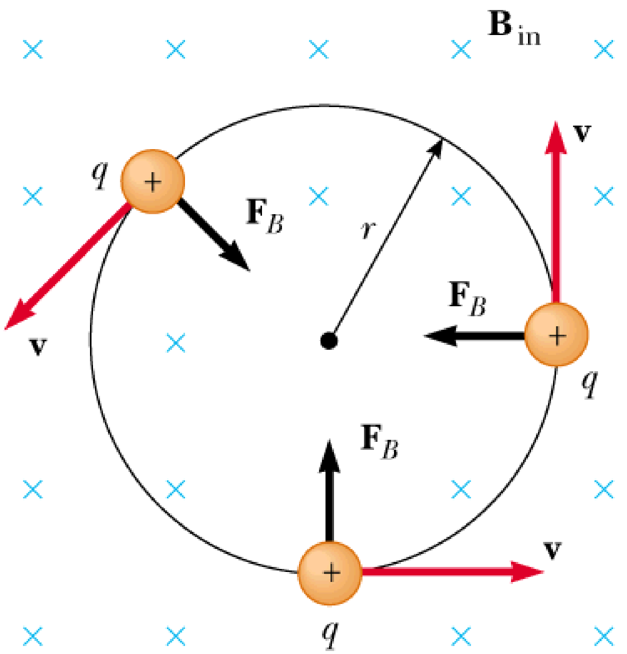
$$\vec{\tau} = I\vec{A}\times\vec{B}$$

$$\vec{\mu} = I\vec{A}$$

$$\vec{\tau} = \vec{\mu}\times\vec{B}$$

$$U = -\vec{\mu}.\vec{B}$$

Cyclotron Motion



$$F_B = qvB = m\frac{v^2}{r}$$

$$r = \frac{mv}{qB}$$

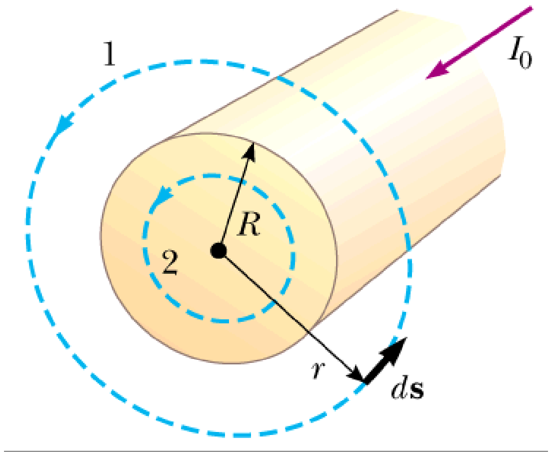
$$\omega = \frac{v}{r} = \frac{qB}{m}$$ → Cyclotron frequency

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

Sources of Magnetic Field

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$

$$\oint \vec{\mathbf{B}}.d\vec{\mathbf{s}} = \mu_0 I$$

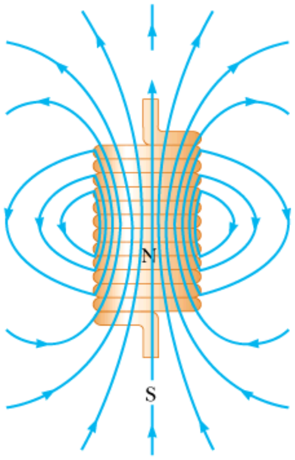


For *r* >= *R*

$$B = \frac{\mu_0 I}{2\pi a}$$

For r < R

$$B = \left(\frac{\mu_0 I_0}{2\pi R^2}\right) r$$



$$B = \mu_0 n I$$

Magnetic Flux and Induction

Magnetic Flux

$$\Phi_B = \int \vec{\mathbf{B}}.d\vec{\mathbf{A}}$$

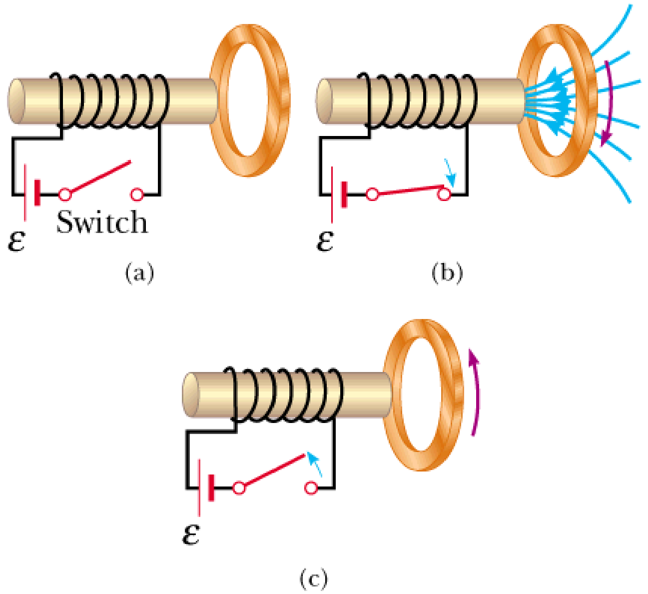
$$\oint \vec{\mathbf{B}}.d\vec{\mathbf{A}} = 0$$

Faraday's Law

$$\mathcal{E} = -N\frac{d\Phi_B}{dt}$$

$$\oint \vec{\mathbf{E}}.d\vec{\mathbf{s}} = -N\frac{d\Phi_B}{dt}$$

The polarity of the induced emf is such that it tends to produce a current that creates a magnetic flux to oppose the change in magnetic flux through the area enclosed by the current loop.



Maxwell's Equations

$$\oint \vec{\mathbf{E}}.d\vec{\mathbf{A}} = \frac{Q}{\varepsilon_0}$$

Gauss' Law

$$\oint \vec{\mathbf{B}}.d\vec{\mathbf{A}} = 0$$

Gauss' Law for Magnetism – no magnetic monopoles

$$\oint \vec{\mathbf{E}}.d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$$

Faraday's Law

$$\oint \vec{\mathbf{B}}.d\vec{\mathbf{s}} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

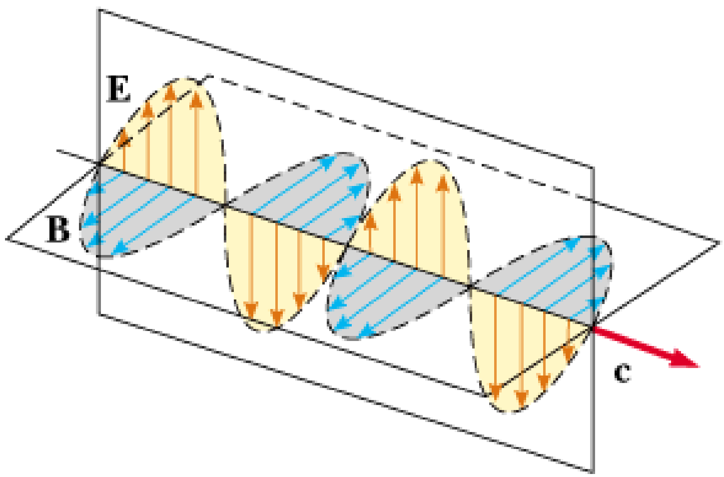
Ampère-Maxwell Law

Displacement Current

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

Lorentz Force Law

EM Waves



$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$ Speed of Light

$$\omega = 2\pi f$$ Angular Frequency

$$\lambda = \frac{c}{f}$$ Wavelength

$$k = \frac{2\pi}{\lambda}$$ Wavenumber

$$E = E_{\max} \cos(kx - \omega t)$$

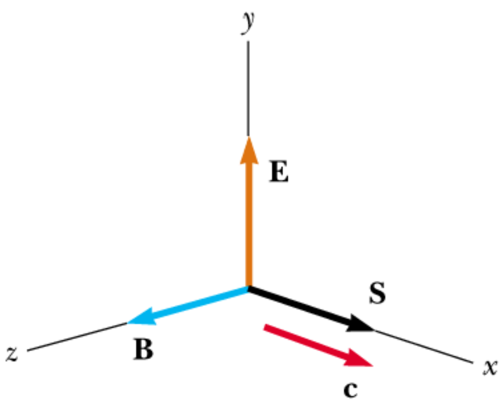
$$B = B_{\max} \cos(kx - \omega t)$$

$$\frac{E}{B} = \frac{E_{\max}}{B_{\max}} = \frac{\omega}{k} = c$$

EM Energy, Momentum and Pressure

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}}$$

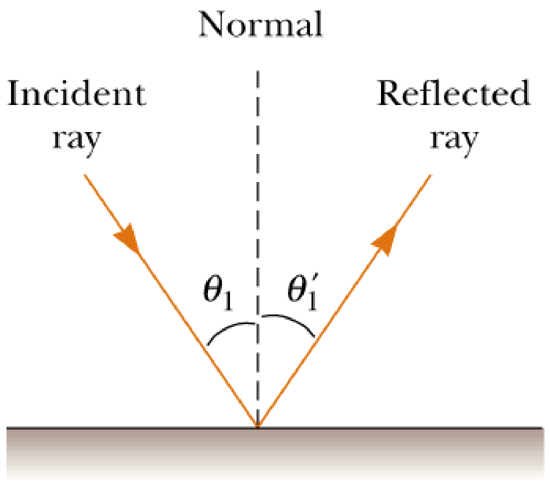
$$I = S_{av} = c u_{av}$$

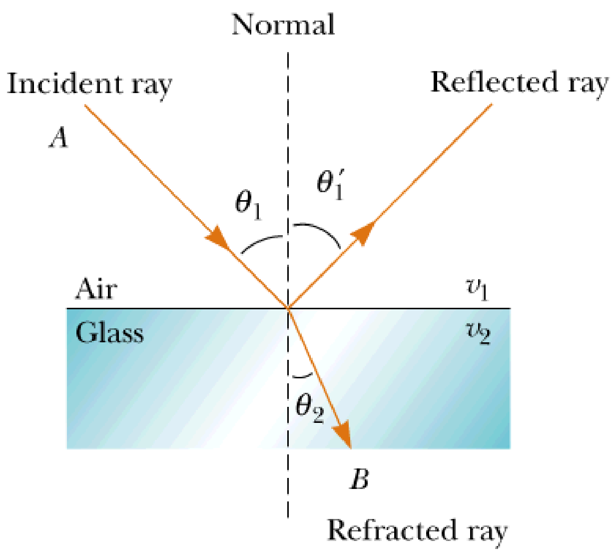


$$u_{av} = \varepsilon_0 \left(E^2\right)_{av} = \frac{1}{2}\varepsilon_0 E_{\max}{}^2 = \frac{B_{\max}{}^2}{2\mu_0}$$

	Momentum	Pressure
Complete Absorption	$p = \frac{U}{c}$	$P = \frac{F}{A} = \frac{1}{A}\frac{dp}{dt} = \frac{S}{c}$
Complete Reflection	$p = \frac{2U}{c}$	$P = \frac{2S}{c}$

Reflection and Refraction





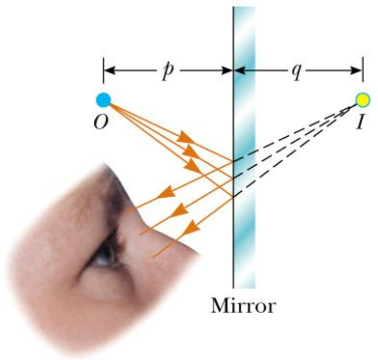
$$\theta_1' = \theta_1$$

$$n = \frac{c}{v}$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

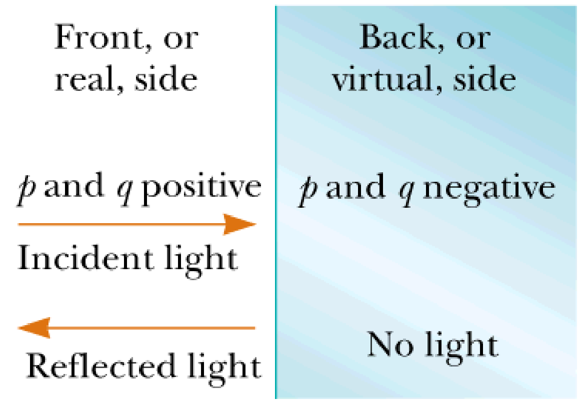
$$\sin\theta_c = \frac{n_2}{n_1}$$

Images by Mirrors

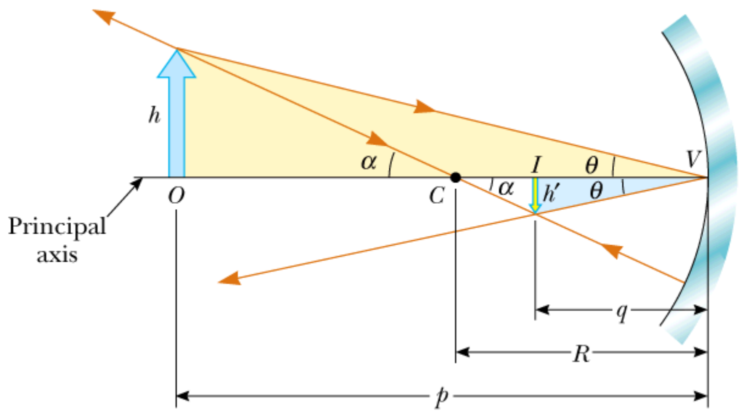


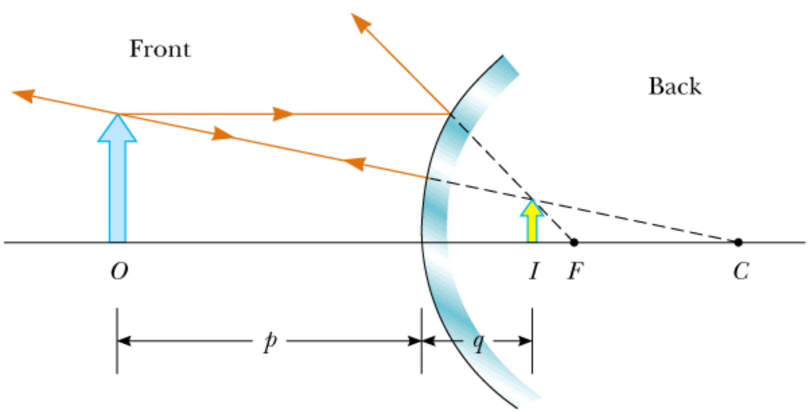
$$p = q$$

$$M = 1$$



Convex or concave mirror



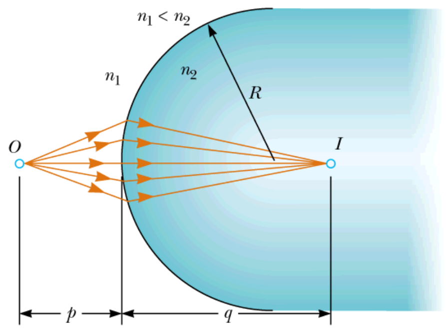


$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

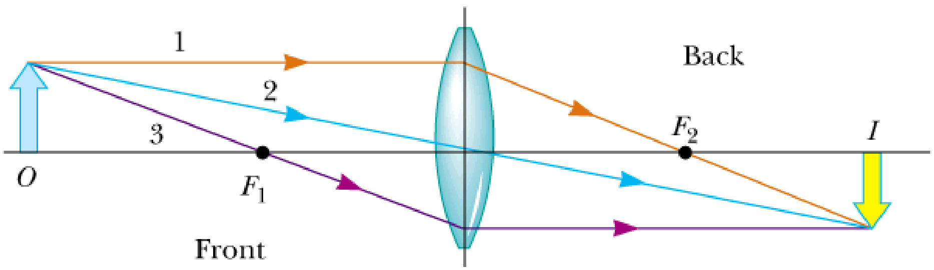
$$f = \frac{R}{2}$$

$$M = \frac{h'}{h} = -\frac{q}{p}$$

Images by Refraction and Lenses



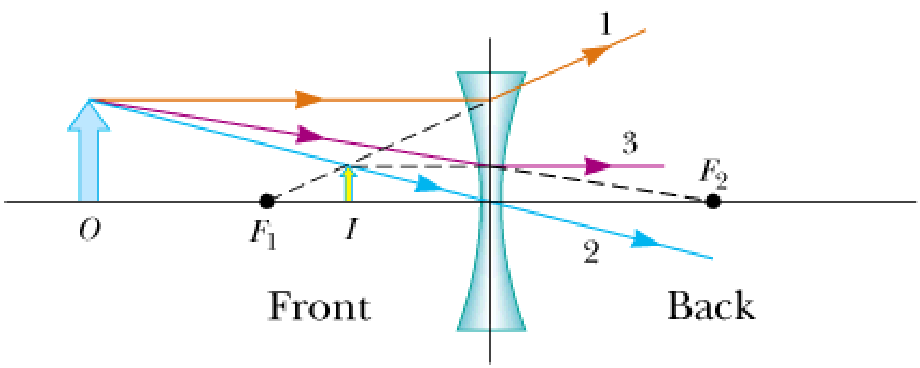
$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{(n_2 - n_1)}{R}$$

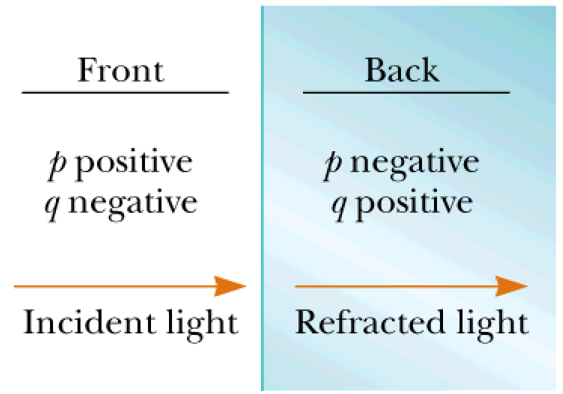


$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$M = \frac{h'}{h} = -\frac{q}{p}$$





Interference

- The intensity observed at any point is proportional to the time average of the sum of the fields incident on that point.
- If the fields are coherent and monochromatic they can interfere with each other, creating bright and dark areas (fringes).
- Destructive or constructive interference depends on the relative phase of the waves with each other.

$$\phi = 2\pi \frac{\delta}{\lambda}$$

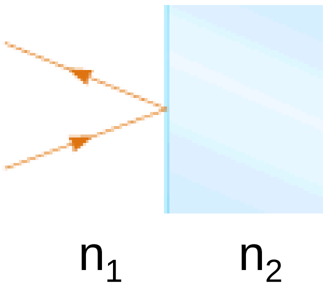
Causes for Phase Difference

Path length difference

$\delta = d \sin\theta$ Double Slit

$\delta = 2(nt)$ Thin Film

Reflection

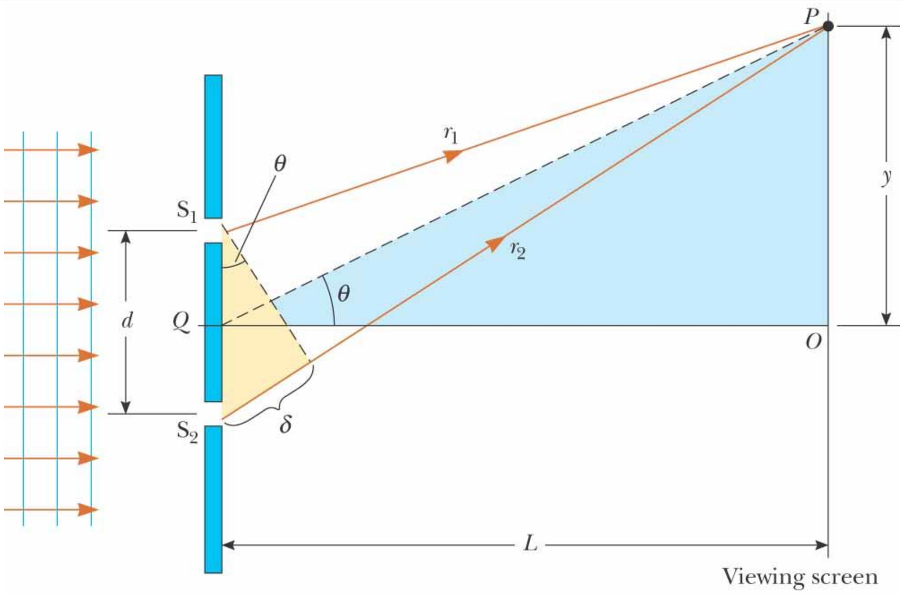


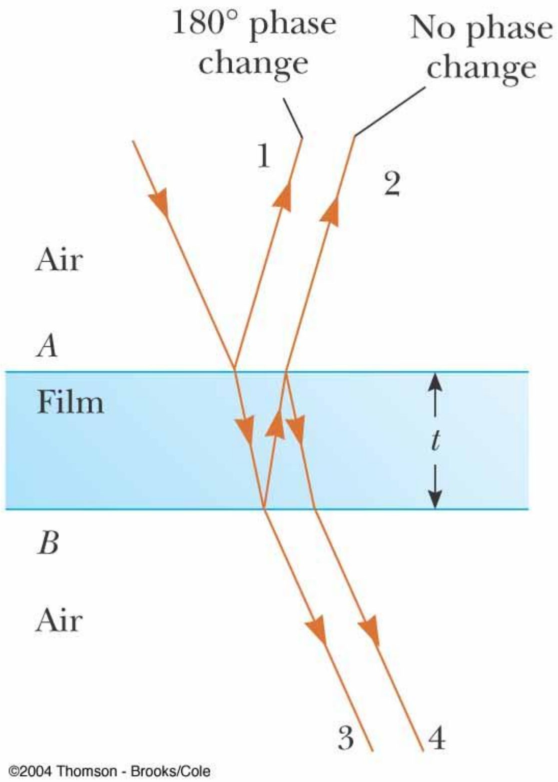
$n_1 > n_2$: no phase

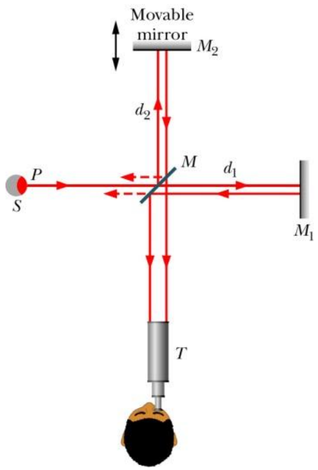
$n_1 < n_2$: π phase

δ	Relative phase change on reflections	
	0	π
$m\lambda$	Const.	Dest.
$(m+½)\lambda$	Dest.	Const.

The Geometries



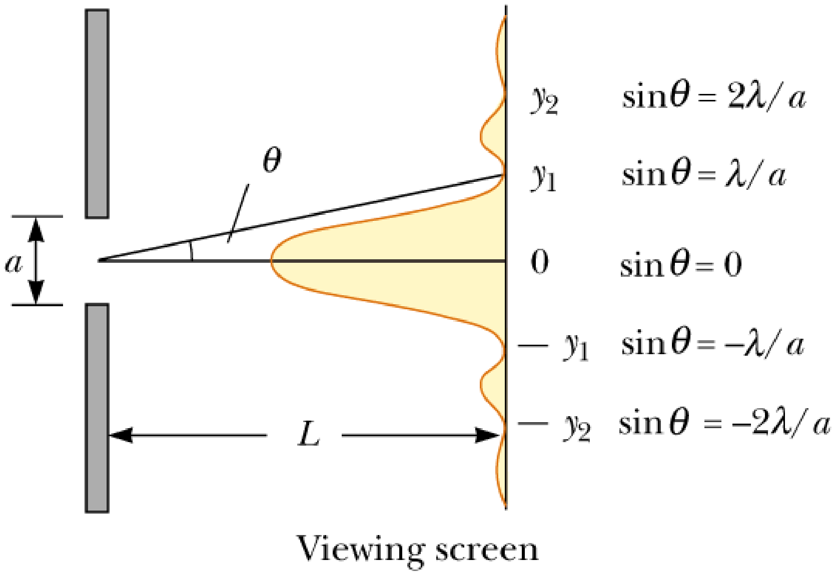




Diffraction

- Rays going through obstacles or openings can diffract around the edges.
- The amount of diffraction depends on the ratio of the wavelength of the wave to the size of the opening/obstacle.
- The diffracted beam interferes with itself to create a pattern of bright and dark fringes on a far away screen.

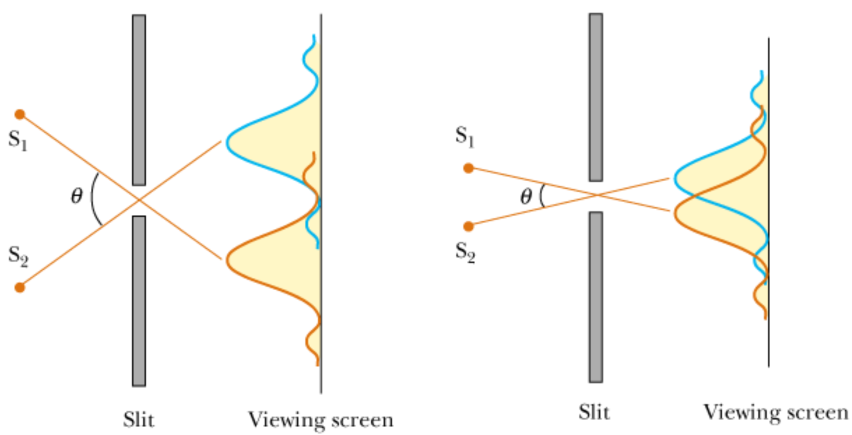
Narrow Slit Diffraction



$$\sin\theta = \frac{m\lambda}{a}$$

$$y_m \approx \frac{L}{a}\lambda m$$

$$m = \pm1, \pm2, \pm3, \ldots$$



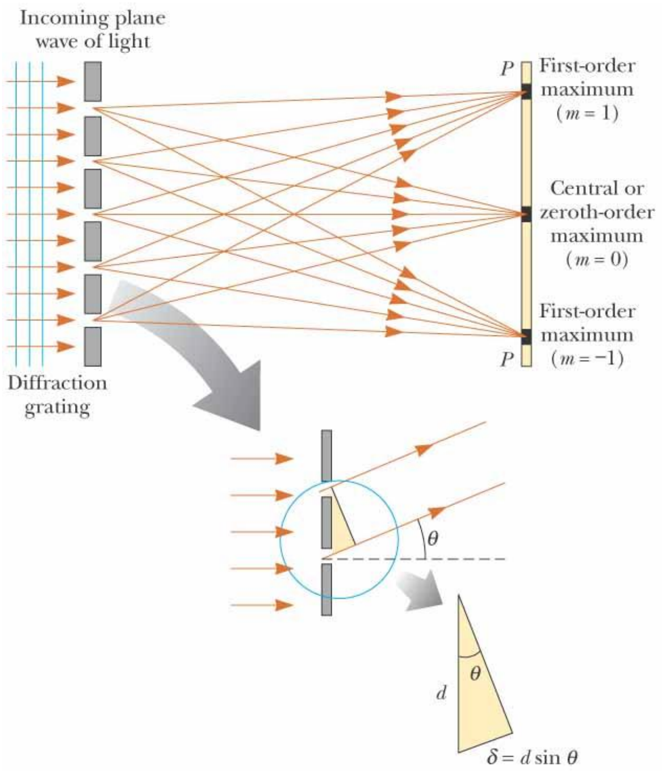
$$\theta_{\min} = \frac{\lambda}{a}$$

For circular apertures

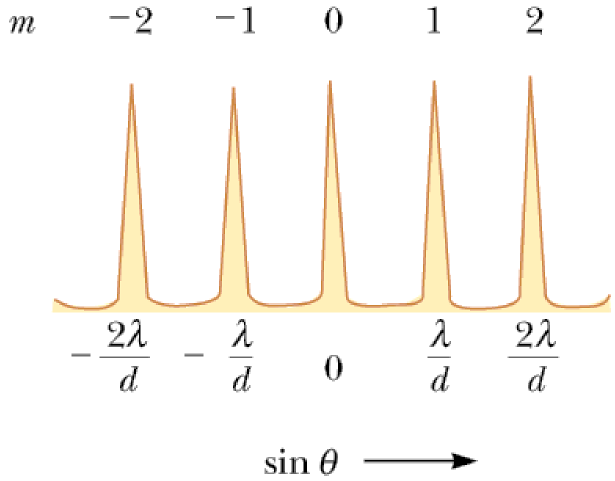
$$\theta_{\min} = 1.22\frac{\lambda}{D}$$

Diffraction Gratings

A device for spectrally analyzing light sources

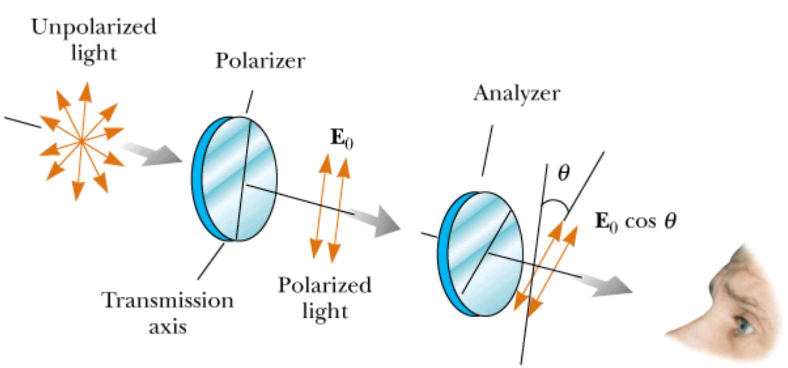


$$d\sin\theta = m\lambda$$

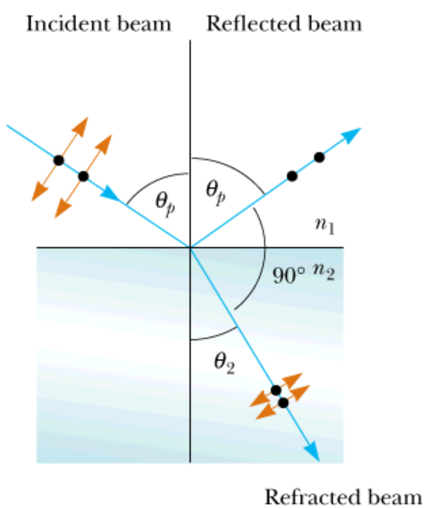


Polarization of Light Waves

- Polarization of a wave is the direction of its electric field vector.
- Ordinary light is unpolarized.
- A polarizer passes light only with polarization along its transmission axis.
- If light is incident on a surface at Brewster's angle, the reflection is also polarized.



$$I_0 \rightarrow I_0/2 \rightarrow (I_0/2)\cos^2\theta$$



$$\theta_p + \theta_2 = 90°$$

$$n = \tan\theta_p$$